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Optimization of power coefficient on a horizontal axis wind turbine using bem theory

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ABSTRACT

Aerodynamic optimization has widely become a issue of considerable interest to determine the geometry of an aerodynamic configuration amidst certain design constraints. Aerodynamic performance is calculated from a prescribed geometric shape, which is often performed in trial and error method. Numerous design methods are available for the aerodynamic design of the rotor. The goal in optimizing is to maximize the aerodynamic efficiency at a single design wind speed. However, single-design point methods do not automatically lead to the optimum design, since they consider only one point in the total operational range. Moreover they do not implicitly involve considerations on loads which require an experienced designer. The aerodynamic optimization of a Horizontal Axis Wind Turbine is a complex method characterized by numerous trade-off decisions aimed at finding the optimum overall performance. However researcher design the wind turbine is an enormous ways and more often decision-making is very difficult. Commercial turbines have been derived from both theoretical and empirical methods, but there is no clear evidence on which of these is optimal. Turbine blades are optimized with the aim to achieve maximum power coefficient for the given blade with solidity, ratio of coefficient of drag to lift, angle of attack and tip speed ratio. In this article, the blade element theory is used to find the optimum value analytically. The effect of power coefficient for different blade angle, tip speed ratio, ratio of coefficient of drag and coefficient of lift and blade solidity is presented and the optimized set value is obtained.

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1. Introduction

Wind is caused due to uneven heating of the land and water by the sun on the earth. This temperature difference induces circulation of air from one region to another. Wind energy conversion system is one with low investment and high yield power generation. The main advantage of electricity generation from wind energy is the absence of harmful emissions. Thirst on energy, the missile price increase of fossil fuels, hazardous impact on environment, uncertain supply of fuel have made the nation to rely on renewable energy sources. Variation in wind velocity round the clock must be extracted with the available wind electric converter and hence wind electric generators must be capable to operate under varying speed and this led to the development of variable-speed wind turbines nowadays. The power produced by a wind electric generator at a specific site depends upon the mean wind speed at the site, hub height, the speed characteristics of the wind turbine cut-in speed, rated speed, and furling wind speeds.

2. Performance and design aspects of horizontal axis wind turbine

Designing wind turbines to achieve satisfactory levels of performance and durability should have deep knowledge in the factors affecting wind power, aerodynamic forces acting in the turbine. Energy conservation, pollution prevention, resource efficiency, systems integration and life cycle costing are very important terms for sustainable construction. Designing wind turbine principles includes: (i) minimizing non-renewable resource consumption, (ii) enhancing the natural environment and (iii) eliminating or minimizing the use of toxins, thus combining energy efficiency with the impact of materials on occupants [1]. Therefore, possible use of wind energy must be evaluated in terms of its impact on the environment.

2.1. Power available in the wind

The three factors that determine the output from a wind energy converter are wind speed, cross-section of wind swept by rotor, and overall conversion efficiency of the rotor, transmission system and generator. Energy available in wind is equal to the kinetic energy of wind. If ρ is the density of the air in kg/m^3, A is the swept area in m^2, and V_m is the mean velocity of wind in m/s, then total wind power available, P_a,

$$P_a = \frac{m_w V_m^2}{2} Watts = \frac{\rho A V_m^3}{2} Watts$$

The above equation shows that the wind power varies as the cube of the wind velocity. However density varies with pressure, temperature and relative humidity. Unfortunately, the total wind energy cannot be recovered in a wind turbine because the output wind velocity cannot be reduced to zero, otherwise there would be no flow through the turbine [2]. Let V_i be the inlet wind velocity and V_o be the outlet wind velocity, $m_w = \rho \cdot \mathrm{A} \cdot V_{ave}$ be the mass flow rate with average velocity $(V_i+V_o)/2$ and the power recovered from the wind (P_{out}) is equal to the rate of change in the kinetic energy.

$$\begin{aligned} P_{out} &= m_w(V_i^2 - V_o^2)/2 \\ &= \rho A V_{ave}(V_i^2 - V_o^2)/2 \\ &= (\rho A)(V_i + V_o)/2(V_i^2 - V_o^2)/2 \\ &= (\rho A/4)(V_i^3 + V_i^2 V_o - V_i V_o^2 - V_o^3)/V_i^3 \end{aligned}$$

$$Take\ x = V_o/V_i$$

$$P_{out} = (1 + x - x^2 - x^3)P_a/2 \quad (1)$$

Differentiating (1) with respect to x and equating to zero, we get the optimum value of x for maximum power output.

$$d(P_{out})/dx = 0 \Rightarrow 1 - 2x - 3\times^2 = 0$$

Solving the quadratic equation, the value of $x = 1/3$.
Substituting the value of x in (1), we get

$$P_{out\ \mathrm{max.}} = 0.593 P_a$$

Thus the maximum that can be drawn from the wind system is 59.3% of the total wind power available, which is called Betz limit in aerodynamics.

The power coefficient C_p is defined as

$$C_p = \frac{P_s}{(1/2).\rho.\pi.R^2.U_o^3} \quad (2)$$

where, P_s is the shaft power output in Watts, U_o is the upstream undisturbed wind speed in m/s. The power performance of a wind turbine can be expressed using fixed angular velocity. This parameter is defined as

$$C_M = \frac{C_p}{\lambda} \quad (3)$$

Wind turbines have various C_p values depending on the wind velocity. Therefore, their efficiency is best represented by a C_p–k curve. The tip speed ratio, λ, is given by

$$\lambda = \frac{\omega R}{V} \quad (4)$$

where λ is tip speed ratio, R is maximum rotor radius (m), ω is rotor speed (rad/s) and V is wind velocity (m/s).

The available wind energy, E_a in the time period T is given by [3]

$$E_a = \int_T P_a dt = \frac{1}{2}\rho A \int_T V_m^3 dt = \frac{1}{2}\rho A V_m^3 = E_{as} A \quad (5)$$

where $V_m{}^3$ and E_{as} are the cubic mean wind speed and the available energy flux in the period T.

2.2. Electrical power output

The power in the wind is converted into mechanical power with power coefficient C_p, with generator efficiency η_g, and mechanical power transmission efficiency η_m, then the electrical power output P_e is given as

$$P_e = C_p . \eta_g . \eta_m . P_a \text{ Watts} \quad (6)$$

Optimum values of C_p, η_g and η_m are 0.45, 0.9 and 0.95 respectively which give an overall efficiency of 38%. Actual values will probably range between 25 and 30% [2] which may vary with wind speed, type of turbine and the nature of load. As the wind increases from a low value, the turbine overcomes all mechanical and electrical losses and start delivering electrical power to the load at cut – in – speed V_C. Rated output power will reach at rated wind speed V_R, above which constant power output is maintained. At the furling speed V_F, the machine is shut down to protect it from high winds.

The efficiency of a wind turbine is usually characterized by its power coefficient as given below. Maximum values of C_p can be 0.5926 according to Betz criteria [4].

$$C_{pe} = \frac{I.V}{\eta_{mech} . \eta_{alternator} . (1/2) . \rho . R^2 . V^3} \quad (7)$$

2.3. Blade design

2.3.1. Aerodynamic design

Blade design consists of aerodynamic and structural design. The challenge is designing rotor blades for different applications with optimised weight and aerodynamic performance. The major applications of rotor blades are: (i).Stall control with constant speed, (ii). Stall control with variable speed, (iii). Pitch control with constant speed, (iv). Pitch control with variable speed. The usage of advanced design tools, production technology and material choice are the dominating parameters. As the power curve is based on the $C_p - \lambda$ characteristics of a certain rotor blade, the main parameters for obtaining an optimum power curve is the rotor diameter, rotational speed and pitch angle [5]. The design of the windmill blade depends on the following parameters: diameter of windmill D, aerofoil characteristics and the number of blades, Z [6].

2.3.2. Production technology

With composite material, production technology influences significantly the design. In the field of rotor blade production, the traditional hand lay-up procedure using polyester and/or epoxy resin as matrix material together with glass fibres will be substituted in the near future by more advanced technologies. The application of preimpregnated material usually suffer from too high material costs, as it is more economic to use the raw materials itself in the production, thus using resin and fibres. An on-line impregnation technology during the production is to be used for an adequate production of unidirectional stiffeners. The usage of raw materials, such as filaments of glass fibre, is generally said cheaper than using fabrics with different lay-up combinations. Therefore, a compromise has to be found between the usage of the different raw materials and its consequences in design, especially in the flexibility of the structure.

2.3.3. Material properties

Basically, there exist four material groups used for rotor blades: Epoxy resin/glass fibre, Polyester resin/glass fibre, Epoxy resin/wood, Epoxy resin/carbon-glass fibres. Further improvements in the material choice, such as using carbon fibres in a hybrid system together with glass fibres has been used for rotor blades larger than 25 m, as a sufficient bending stiffness is required [7]. Combined with an optimised structural design and thick profiles, it is also possible to use only glass fibres for rotor blade with a length of 30 m. The weight/strength ratio is the driving parameter for the determination of the optimum material and lay-up combination. Sandwich structures with foams are necessary for the structural stability. On the other hand, material damping is one of the mayor issues concerning the dynamic behaviour of the complete system rotor blades-wind turbines, especially for epoxy/glass fibre and polyester resin/glass fibre systems. The material wood, combined with epoxy resin, seems theoretically to have excellent performance. Most of the wind turbine blades are made of fiberglass reinforced with polyester or epoxy resin. Small turbine blades are made of steel or aluminium, but the drawback is huge weight. Lighter and more effective blades decrease material requirements for other wind turbine component making overall costs to be lower. Materials with lower density such as fiber aramid (Technora) have higher natural frequencies and bigger deflection. Blades made of fiberglass can be reconstructed with carbon based composites to reduce mass and increase its stiffness.

Suggestions for increasing performance and safety of windmill systems listed by Onder Ozgener [4] are as follows.

- Blades can be made of epoxy–carbon fiber or glass fiber reinforced plastics.
- To produce a smooth surface a steel mold can be used.
- A long and narrow airfoil can be selected having larger aspect ratio than the classical (short and wide wing) blade.
- Steel blades should not be used due to their weight and corrodibility.
- Lighting protection can be provided for GRP epoxy–carbon fiber blades.
- There is no requirement that the same profile should be used throughout the blade length.

2.3.4. Weight

The blade mass is one of the most important parameters for dynamic loads of blade and wind turbine. The aim is to achieve an optimum between a low weight blade, related to low-cost production and a high performance. The blade weight can be reduced by thick profiles, thus increasing the moment of inertia of the blade cross section. This allows, taking into account the material elastic properties, a high bending stiffness.

2.3.5. Noise

The major sources of noise emission of rotor blades are (i). Turbulence in flow noise, (ii). Trailing edge, (iii). Tip. The aerodynamic lay-out, which aims an optimum aerodynamic

performance, is influenced by the obligation to design low-noise profiles and to adapt the structural lay-out, especially the profile thickness Furthermore, dirt on the blade surface contributes to noise emission.

2.3.6. Lightning protection

The most typical damage due to lightning is at the tip region, where the increasing temperature leads to the build-up of air pressure. Therefore, a metal alloy receptor is integrated in the tip region, and a metal stripe inside the blades transports the energy to the blade root connection. From there, the energy is transported to earth through the turbine structure. With this lightning receptor, it is only necessary to repair the area around the tip.

2.4. Wind statistics

Wind is a highly variable power source, and there are several methods of characterizing this variability. The most common method is the power duration curve which is a good concept but is not easily used to select V_C and V_R for a given wind site, which is an important design requirement. Another method is to use a statistical representation, particularly a Weibull function [2]. Local values of wind velocity should be 3 m/s or higher, and the wind should be steady, to produce electricity effectively.

3. Optimization for wind turbine rotors

This optimization of aeroturbine focuses on the development of multi-disciplinary optimization algorithm for designing of horizontal axis wind turbines with multiple constraints. The aim of the optimization process is to optimum potential reduction in cost, the optimum specific power and the optimum airfoil characteristics. Design variables were rotor chord, twist, relative thickness and structural shell thickness along the blades with the tip pitch angle [7].

3.1. Optimum cost of energy

To compensate the reduction in annual energy production, the swept area can be increased to gain energy yield, without increasing generator size and design fixed loads and hence total cost. It would be possible to constrain the energy yield to a minimum acceptable value.

3.2. Optimum rotor geometry

The optimization of rotor geometry returns smooth shapes. On reducing the chord, the blade weight, extreme loads and fatigue loads are reduced from the reduction in projected blade area. The twist in the root region is of minor importance to the power.

3.3. Optimum airfoil characteristics

To investigate the optimum airfoil characteristics, the lift and drag characteristics should be considered as design variables. The high at the root is often studied because of increased production at low wind speeds before rated power. A reduction in chord reduces both blade weight and extreme loads, but should be counterbalanced by an increase in the $C_{L.\ max}$ to maintain power.

3.4. Cost of energy versus specific power

Optimizations were done with different constraints on the maximum generator power to investigate the variation of cost of energy with the specific power. Optimum aerodynamic efficiency at some design wind speed is closely related to the rotor shape, however the efficiency depends on the number of rotor.

3.5. Blade properties

The aerodynamic profiles of wind turbine blades have crucial influence on aerodynamic efficiency of wind turbine. When blades of length more than 45 m are used, the dynamic behavior of the blade should be taken into account, and the position and shape of the spars have to be analyzed. The location of main spar together with the location of the stiffness ribs will have the biggest influence on the bending modes of the blade. The twist of spars decides about pitch of principal bending axes.

3.6. Aerodynamic loads

Blade Element Momentum (BEM) method is used for the analysis of aerodynamic loads. It is an iterative method, which assumes the value of axial retardation coefficient ‘a’ to be zero at the beginning. The aerodynamic loads are expressed in the following formulas: (c is the chord of aerodynamic profile)

$$\text{Lift}L, = \frac{1}{2}.\rho.V_{rel}^2.c.C_L \tag{8}$$

$$\text{Drag}, D = \frac{1}{2}.\rho.V_{rel}^2.c.C_D \tag{9}$$

$$\text{Thrust}, F_N = L.\cos\ \phi + D.\sin\ \phi \tag{10}$$

$$\text{Torque}, F_T = L.\sin\ \phi - D.\cos\ \phi \tag{11}$$

3.7. Blade material

The blade is made of composite materials with more than one bonded material with different structural properties to achieve the combination of desirable properties, with the main advantage of high ratio of stiffness to weight. One of the materials, reinforcing phase is embedded with the other material, matrix phase. The special care must be taken in defining the properties and orientations of the various layers since each layer may have different orthotropic material properties. Carbon fiber composites allow to less blade mass.

4. Composite rotor blades

The primary objective of composite rotor blades is to minimize the blade weight, subject to frequency and auto rotational inertia constraints. A composite is a structural material which consists of combining two or more constituents. The constituents are combined at microscopic level and are not soluble in each other. One constituent is called the reinforcing phase and the one which is embedded is called the matrix. The reinforcing phase material may be in forms of fibers, particles and flakes. The matrix phase materials are generally continuous. Strength of the composite materials depend on (i).Orientation of the fiber (ii).Type and amount of the fiber present. Fiber orientation in each layer as well as stacking sequence of the plies plays a major role in strength and modulus of the composite laminates. Fibers oriented in one direction will have high strength and stiffness in the direction of orientation.

4.1. Design function requirements

The design functions to be considered are as follows

4.1.1. Stiffness and strength
A combination of high strength and stiffness is desirable because of the vibration from the natural frequencies in the air frame and the periodic loads experienced by the blade.

4.1.2. Weight
The most important fact of using composite material is considerable weight saving which is determined by the mass moment of inertia.

4.1.3. Safety
Predictable and confidence in the material arises only with the realistic safety margins, to maintain safety in the blades.

4.1.4. Impact resistance
The blades should have the ability to resist not only the impact of foreign bodies but also certain level of mishandling during servicing.

4.1.5. Erosion
The erosion materials, particles in the air such as dust, sand are very abrasive in nature. So all the leading edges of the blades should be constructed with abrasive resistant materials.

4.1.6. Corrosion
Corrosion increases the safety margins and decreases the maintenance. So the entire part of the blade should be made of corrosive resistant materials.

4.1.7. Cost
The main design optimization of composite material is to satisfy the cost requirement, i.e., at low cost. The cost includes low initial cost, low operating cost and low maintenance cost.

4.1.8. Endurance
Improving the survival will lead to high reliability and less maintenance. The life of the blades has important implications on operating cost and must be maximized to ensure economic viability.

4.1.9. De-icing
A facility for locally heating the leading edge of the blade is required for de-icing purpose.

4.1.10. Lightning strike protection
If lightning strikes occur, an electrically conductive path is required along the blade length to discharge the high voltage.

5. Momentum theory

Wind Turbine extracts kinetic energy from the wind. Kinetic energy in the wind is absorbed by wind turbine by slowing down the wind. If it is assumed that mass of the air passing through the turbine is separated from the mass that does not passing through the turbine, then the separated part of the flow field remains a long stream tube which lies upstream and downstream of the turbine. Fig. 1 shows the actuator disk model of a wind turbine.

The assumptions made in this theory are (i) the turbine must be a horizontal axis configuration such that an average stream tube can be identified, (ii) The portion of kinetic energy in the swirl component of velocity in the wake is neglected and (iii) the effect of the radial pressure gradient is excluded. The upstream wind velocity V is decelerated to $V(1-a)$ at the turbine disk and to V(1-2a) in the wake of the turbine. Momentum analysis predicts

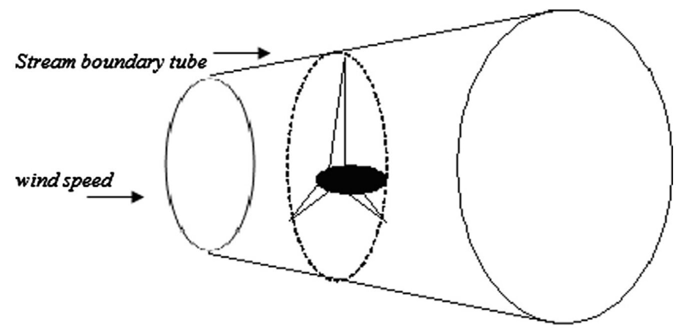

Fig. 1. Actuator disk model of a wind turbine.

the axial thrust on the turbine of radius R to be

$$T = 2\pi R^2 \rho V^2 a(1-a)(\text{in N}) \tag{12}$$

where T is the axial thrust on the wind turbine (N), R is the turbine radius (m), ρ is the air density at sea-level at standard atmospheric conditions (kg/m^3), $V$is the wind speed (m/s), $a$is the axial induction factor.

$$\text{Thrust coefficient } C_T = \frac{T}{0.5\rho\pi R^2 V^2} = 4a(1-a) \tag{13}$$

Mechanical power produced by the turbine is given as P,

$$P = 2\pi R^2 \rho V^3 a(1-a)^2(\text{in W}) \tag{14}$$

Wind power in the upstream wind covering an area equal to rotor disk P_W (in W),

$$P_W = 0.5\rho V^3 \pi R^2 \tag{15}$$

$$\text{Power coefficient}, Cp = \frac{P}{Pw} = 4a(1-a)^2 \tag{16}$$

Maximum value of power coefficient is at $a = 1/3$, substituting the value of 'a' in Eq. (16), $C_{p\max} = 0.593$

$$\text{Torque coefficient}, C_Q = \frac{Q}{0.5\rho\pi R^2 V^2}$$

where Q is the turbine torque.

$$\text{Also}, C_Q = \frac{C_P}{(U/V)}. \tag{17}$$

6. Blade element momentum theory

Blade-element theory helps to analyze the relationship between the individual airfoil properties and axial induction factor, power produced and the axial thrust of the turbine.

The elemental torque which acts on all blade elements in an annular ring is

$$dQ = 0.5Bcr\rho W^2(C_L \sin\phi - C_D \cos\phi)dr \tag{18}$$

where B is the number of blades, $c$is the chord (m), $r$is the radius of blade element (m), W is the velocity of the wind relative to the airfoil (m/s), C_L is the lift coefficient and C_D is the drag coefficient, ϕ is the flow angle.

The sectional lift and drag coefficients are obtained from empirical airfoil data and are unique functions of the local flow angle of attack and the local Reynolds number of the flow. If $dL$and dD are the lift and drag forces on the blade element respectively, then lift and drag coefficients are defined as

$$dL = C_L(0.5\rho W^2)c.dr \tag{19}$$

$$dD = C_D(0.5\rho W^2)c.dr \tag{20}$$

Power = torque x turbine angular velocity, which can be obtained by integrating Eq. (18) and multiplying the same with angular velocity of turbine.

$$P=0.5\rho B\Omega\int_0^R cW^2(C_L\sin\phi-C_D\cos\phi)dr \tag{21}$$

Similarly, total thrust force on the turbine is given by

$$T=0.5\rho B\int_0^R cW^2(C_L\cos\phi+C_D\sin\phi)dr \tag{22}$$

From the Fig. 2,

$$\sin\phi=V(1-a)/W \tag{23}$$

$$\cos\phi=(1+a')r/W \tag{24}$$

where a' is the tangential induction factor $=\omega/2\Omega$

$$W=[V^2(1-a)^2+(1+a')^2\Omega^2r^2]^{0.5} \tag{25}$$

$$\sin\alpha=\sin(\phi-\theta) \tag{26}$$

For flat plate airfoil, $C_L=2\pi\sin\alpha$ (27)

For symmetric airfoil $C_L=2\pi\alpha$ (28)

For circular arc airfoil, $C_L=2\pi[\alpha+(2f/c)]$ (29)

where f is the maximum thickness of circular arc airfoil (m)

The assumption made in this work is maximum thickness of a circular-arc airfoil is assumed to be 6% of the chord, hence the lift coefficient equation will be

$$C_L=2\pi[\alpha+(0.12)] \tag{30}$$

Power produced by the wind turbine with flate symmetric airfoil plate can be obtained by substituting Eq. (23)–(27) in Eq. (10). i.e.,

$$\begin{aligned}P=\pi\rho B\Omega\Bigg\{&\int_0^R V^2.c.(1-a)^2.c.\cos\theta.r.dr\\&-\int_0^R V.(1-a).(1+a').c.\Omega.\sin\theta.r^2.dr\\&-\int_0^R V.(1-a).(1+a').c.\Omega(C_D/C_l)..\cos\theta.r^2.dr\\&+\int_0^R(1+a')^2.c.\Omega^2(C_D/C_l).\sin\theta.r^{3.dr}\Bigg\}\end{aligned} \tag{31}$$

Simplifying assumptions made for integrating eq. (31) are

i) Uniform distribution of upstream wind speed along the blade
ii) Constant chord along the blade (parallel plan form blade)
iii) Constant blade angle during steady state operation
iv) Constant drag-to-lift coefficient ratio
v) Uniform distribution of axial and tangential induction factors along the blade.

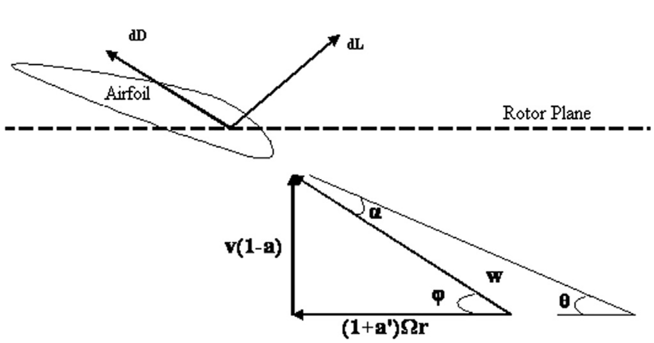

Fig. 2. forces acting on the blade.

Introducing the definitions of solidity and power coefficient and integrating the above Eq. (20) for obtaining power coefficient,

$$\begin{aligned}C_P=\pi\sigma(1-a)(U/V)\{\Bigg\{&[1-a-(2/3)(1-a')(C_D/C_L)(U/V)]\cos\theta\\&-\left[\frac{(2/3)(1+a')(U/V)-0.5(C_D/C_L)(1+a')^2(U/V)^2}{(1-a)}\right]\sin\theta\Bigg\}\end{aligned} \tag{32}$$

Equate Eqs. (21) and (5) to get the axial induction factor,

$$\begin{aligned}a=\left[\frac{\pi\sigma(U/V)(1-a)}{4}\right]\Big\{&[1-a-(2/3)(1+a')(C_D/C_L)(U/V)]\cos\theta\\&-\left[(2/3)(1+a')(U/V)-\frac{0.5(C_D/C_L)(1+a')^2(U/V)^2}{(1-a)}\right]\sin\theta\end{aligned} \tag{33}$$

Similarly, to get the total thrust force on the turbine, substitute the Eq. (23)–(27) in Eq. (22)

$$\begin{aligned}T=\pi\rho B\Bigg[&\int V(1-a)(1+a')c\Omega\cos\theta rdr\\&-\int_0^R(1+a')^2c\Omega^2\sin\theta r^2dr+\int_0^R V^2(1-a)^2c(C_D/C_L)\cos\theta dr\\&-\int_0^R V(1-a)(1+a')c\Omega(C_D/C_L)\sin\theta rdr\Bigg]\end{aligned} \tag{34}$$

Integrating Eq. (34) and introducing the definition of solidity,

$$\begin{aligned}T=\rho\pi^2\sigma(1-a)R^2V^2\Big\{&[0.5(1+a')(U/V)+(C_D/C_L)(1-a)]\cos\theta\\&-\left[\frac{(1+a')^2(U/V)^2(1-a)}{3}+0.5(C_D/C_L)(1+a')(U/V)\right]\sin\theta\Big\}\end{aligned} \tag{35}$$

Equate Eqs. (35) and (12) to get the tangential induction factor.

$$\begin{aligned}a'=&\left[\frac{2}{(U/V)}\cos\theta\right]\left(\frac{2a}{\pi\sigma}\right)-0.5(U/V)\cos\theta\\&-(1-a)(C_D/C_L)\cos\theta+\left[\frac{2}{(U/V)}\cos\theta\right]\\&\left[\frac{(1-a)^2(U/V)^2(1-a)}{3}+0.5(1+a')(C_D/C_L)(U/V)\right]\sin\theta\end{aligned} \tag{36}$$

Substituting equations for circular arc airfoil

$$\begin{aligned}P=\pi\rho B\Omega\Big\{&\int_0^R V^2(1-a)^2c\cos\theta rdr-\int_0^R V(1-a)(1+a')c\Omega\sin\theta r^2dr\\&-\int_0^R V(1-a)(1+a')c\Omega(C_D/C_L)\cos\theta r^2dr\\&+\int_0^R(1+a')^2c\Omega^2(C_D/C_L)\sin\theta r^3dr\Big\}+0.12\pi\rho B\Omega\\&\Big\{\int_0^R V^2(1-a)c(1+Ar^2)^{0.5}rdr\\&-\int_0^R V(1-a)(1+a')c\Omega(C_D/C_L)(1+Ar^2)^{0.5}r^2dr\Big\}\end{aligned} \tag{37}$$

Where,

$$A=(1+a')^2\Omega^2/(1-a)^2V^2$$

$$\begin{aligned}C_P=\pi\sigma(1-a)(U/V)\{&[1-a-(2/3)(1+a')(C_D/C_L)(U/V)]\cos\theta\\&-[(2/3)(1+a')(U/V)-0.5(1+a')^2(C_D/C_L)(U/V)^2/(1-a)\sin\theta]\}\\&+[0.2513\sigma(1-a)^4/(U/V)(1+a')^2]x(E^{1.5}-1)\\&-[0.1885\sigma(1-a)^3(C_D/C_L)E^{1.5}/(1+a')]\\&+[0.0942\sigma(1-a)^3(C_D/C_L)E^{0.5}/(1+a')]\\&+[0.0942\sigma(1-a)^4(C_D/C_L)/(1+a')^2(U/V)]\\&x\ln|[(1+a')(U/V)/(1-a)]+E^{0.5}|\end{aligned} \tag{38}$$

where

$$E = 1 + [(1+a')^2 (U/V)^2/(1-a)^2]$$

$$\begin{aligned} a = {} & [0.7854\sigma(U/V)/(1-a)]\{[1-a-(2/3)(1+a')(C_D/C_L)(U/V)]\cos\theta \\ & -[(2/3)(1+a')(U/V)-0.5(1+a')^2(C_D/C_L)(U/V)^2/(1-a)]\sin\theta\} \\ & +[0.0628\sigma(1-a)^2/(1+a')^2(U/V)](E^{1.5}-1) \\ & -[0.0471\sigma(1-a)(C_D/C_L)/(1+a')]E^{1.5} \\ & +[0.0236\sigma(1-a)(C_D/C_L)/(1+a')]E^{0.5} \\ & +[0.0236\sigma(1-a)^2((C_D/C_L))/(1-a)^2(U/V)] \\ & x\ln\left|[(1+a')(U/V)/(1-a)]+E^{0.5}\right| \end{aligned} \tag{39}$$

$$\begin{aligned} T = {} & \rho\pi^2\sigma(1-a)R^2V^2\{[0.5(1+a')(U/V)+(1-a)(C_D/C_L)]\cos\theta \\ & -[((1+a')^2(U/V)^2/3(1-a)) \\ & +0.5(1+a')(C_D/C_L)(U/V)]\sin\theta\} \\ & +[0.1257\pi\rho\sigma RV^3(1-a)^3/(1+a')\Omega] \\ & +0.1885\pi\rho\sigma R^2V^2(1-a)^2(C_D/C_L)E^{0.5} \\ & +[0.1885\pi\rho\sigma RV^3(1-a)^3(C_D/C_L)/(1+a')\Omega]E^{1.5} \\ & -1x\ln|[(1+a')(U/V)/(1-a)]+E^{0.5}| \end{aligned} \tag{40}$$

$$\begin{aligned} a' = {} & [1.27324/\sigma(U/V)\cos\theta]\{a-0.7854\sigma(U/V)\cos\theta \\ & -1.5708\sigma(C_D/C_L)(1-a)\cos\theta \\ & -1.5708\sigma[((1+a')^2(U/V)^2/3(1-a)) \\ & +0.5(C_D/C_L)(1+a')(U/V)]\sin\theta \\ & -[0.06285\sigma(1-a)^2/(1+a')(U/V)](E^{1.5}-1) \\ & -0.09425\sigma(1-a)(C_D/C_L)E^{0.5} \\ & -[0.09425\sigma(1-a)^2(C_D/C_L)/(1-a)(U/V)] \\ & x\ln|[(1+a')(U/V)/(1-a)]+E^{0.5}|\} \end{aligned} \tag{41}$$

7. Optimization methodology

The theoretical analysis was performed to investigate the effect and dependence of the various parameters in the wind turbine rotor geometry. The analysis includes the recommended values at specified operating conditions to maximize the power extracted by the wind turbine rotor. In this work, the variation in the parameters is as follows; blade angle (θ) is varied between 0 and 10°; (C_D/C_L) is varied between 0 and 0.10; rotor solidity, (σ) is varied between 0.10 and 0.30; tip speed ratio (U/V) is varied between 2 and 14. The eqs. (33) and (36), since they are coupled

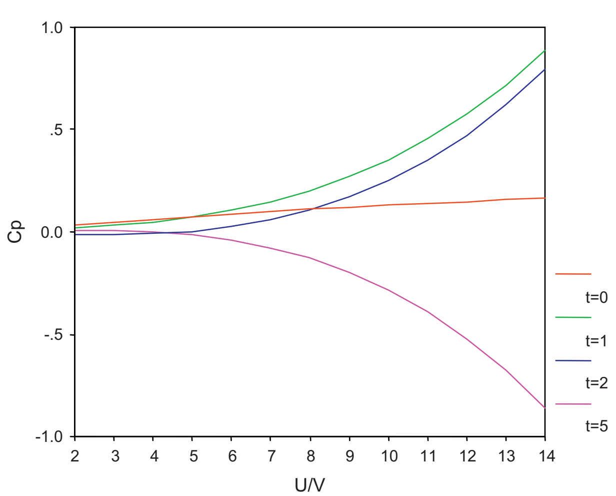

Fig. 3. Effect of power coefficient C_P with tip speed ratio (U/V) for different blade angle θ with (C_D/C_L) ratio of 0.022 and rotor solidity (σ) as 0.01.

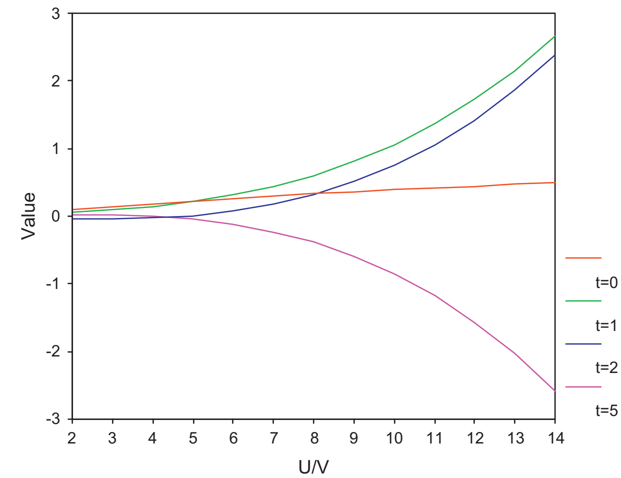

Fig. 4. Effect of power coefficient C_P with tip speed ratio (U/V) for different blade angle θ with (C_D/C_L) ratio of 0.022 and rotor solidity (σ) as 0.03.

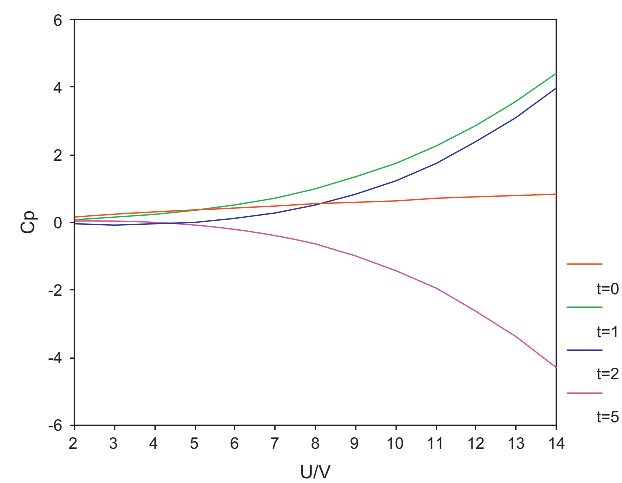

Fig. 5. Effect of power coefficient C_P with tip speed ratio (U/V)for different blade angle θ with (C_D/C_L) ratio of 0.022 and rotor solidity (σ) as 0.05.

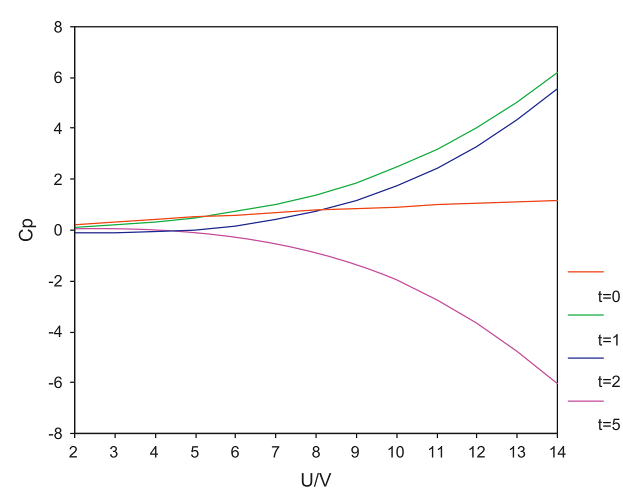

Fig. 6. Effect of power coefficient C_P with tip speed ratio (U/V)for different blade angle θ with (C_D/C_L) ratio of 0.022 and rotor solidity (σ) as 0.07.

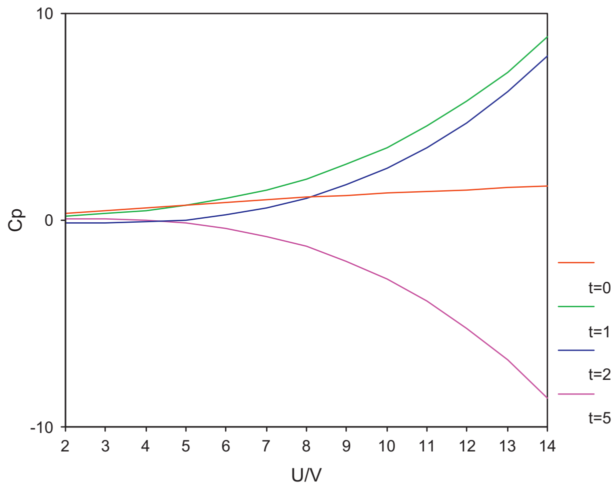

Fig. 7. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.022 and rotor solidity (σ) as 0.1.

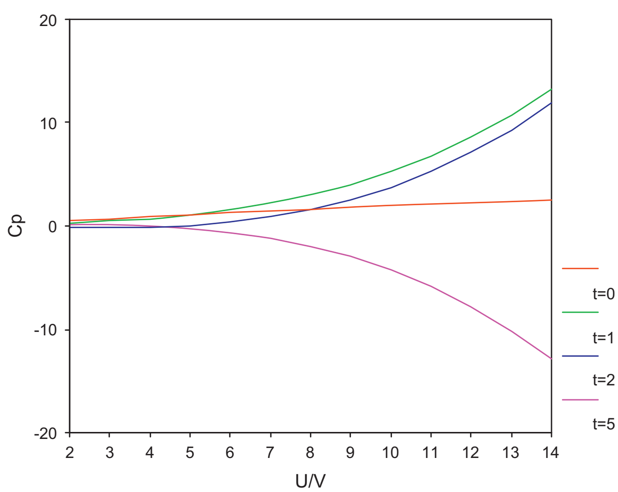

Fig. 8. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.022 and rotor solidity (σ) as 0.15.

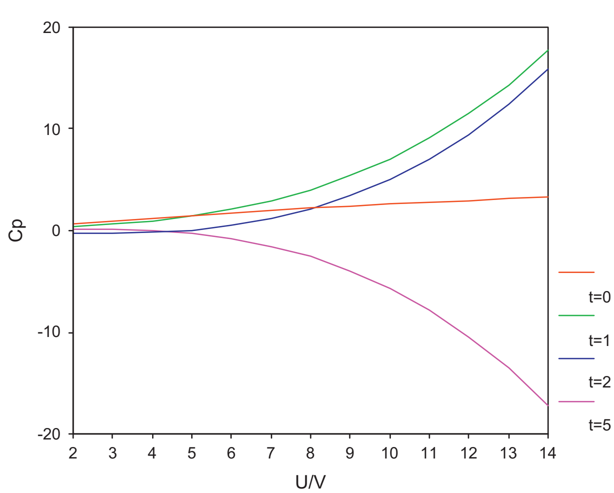

Fig. 9. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.022 and rotor solidity (σ) as 0.2.

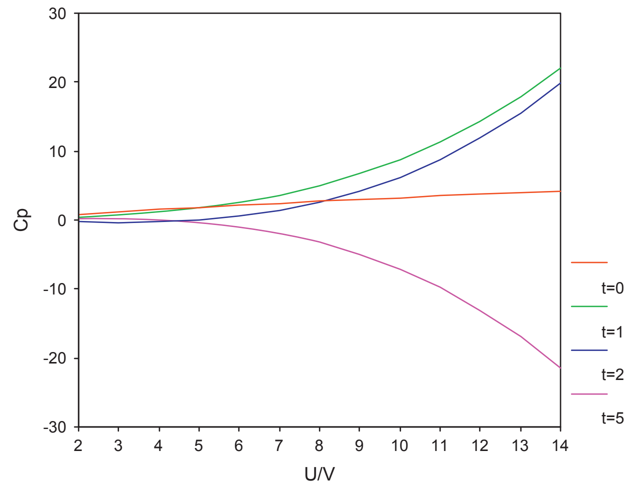

Fig. 10. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.022 and rotor solidity (σ) as 0.25.

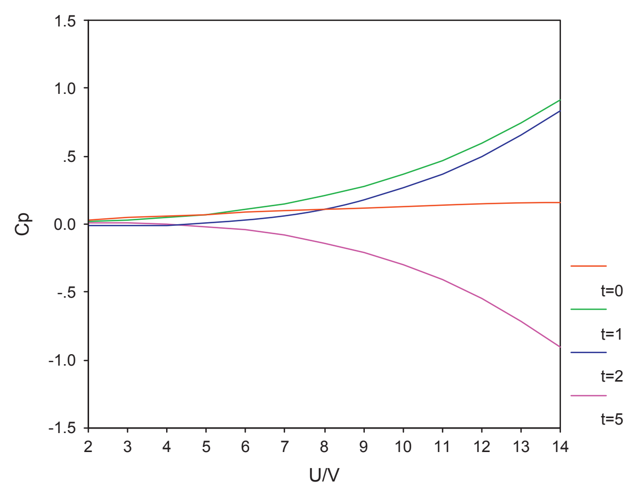

Fig. 11. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.023 and rotor solidity (σ) as 0.01.

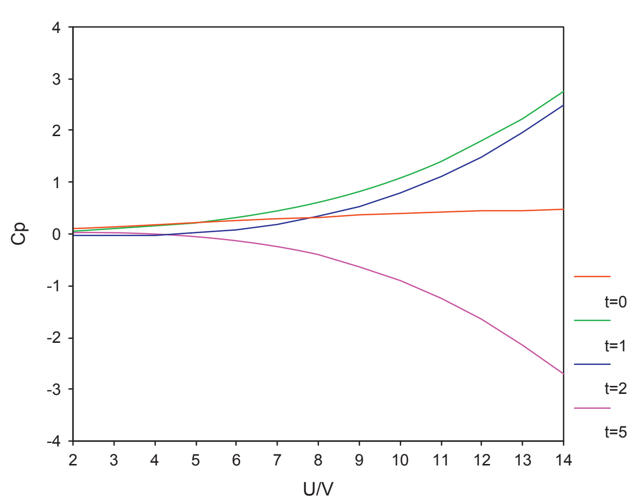

Fig. 12. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.023 and rotor solidity (σ) as 0.03.

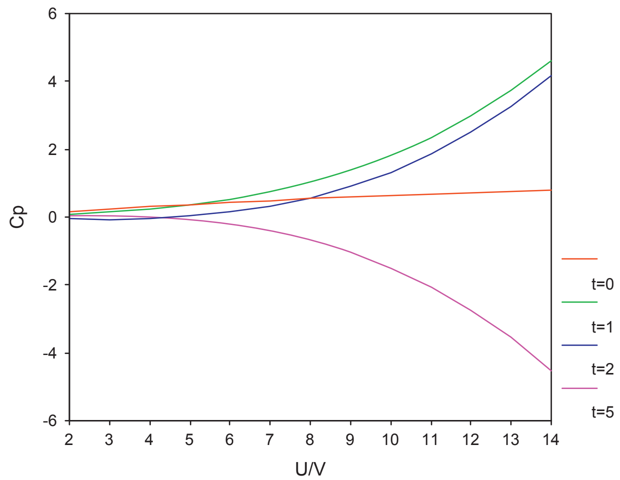

Fig. 13. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.023 and rotor solidity (σ) as 0.05.

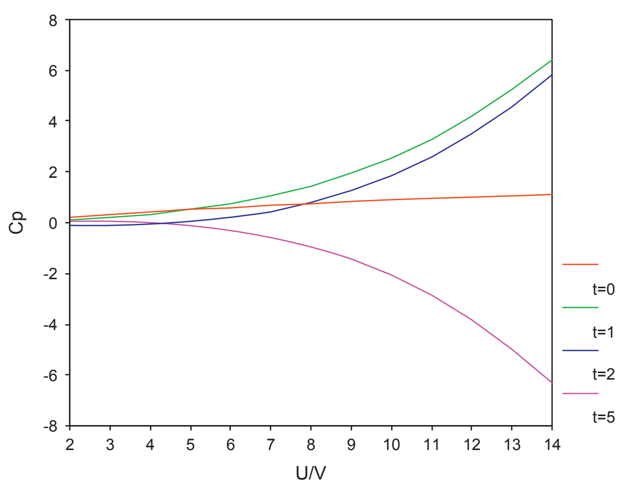

Fig. 14. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.023 and rotor solidity (σ) as 0.07.

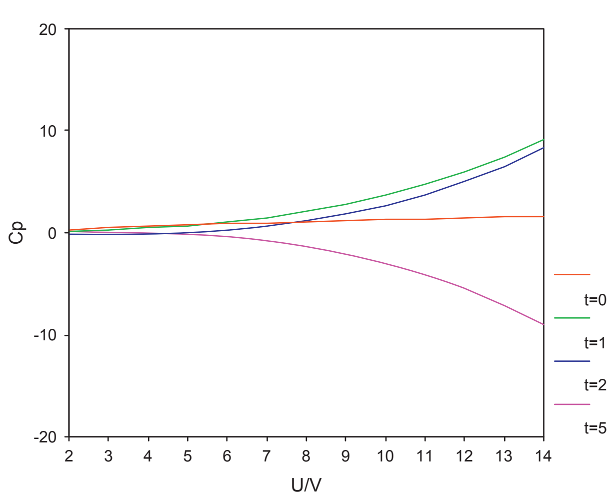

Fig. 15. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.023 and rotor solidity (σ) as 0.10.

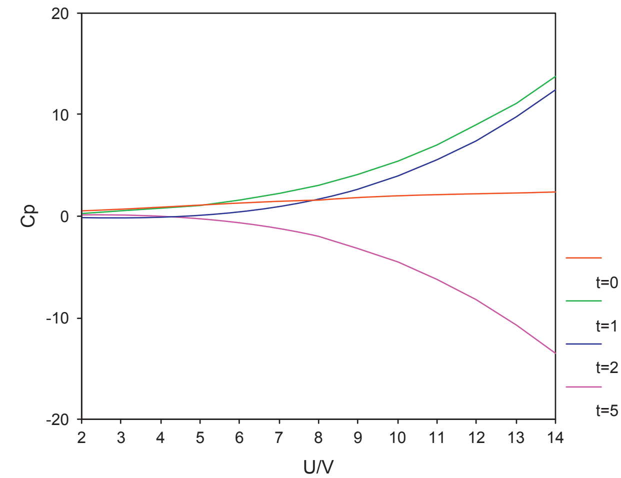

Fig. 16. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.023 and rotor solidity (σ) as 0.15.

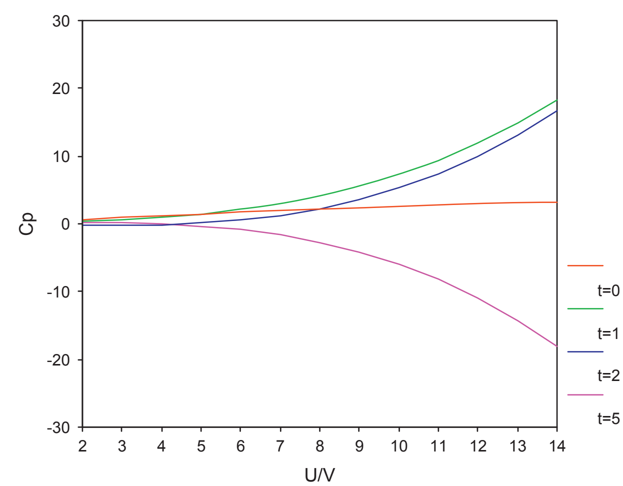

Fig. 17. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.023 and rotor solidity (σ) as 0.20.

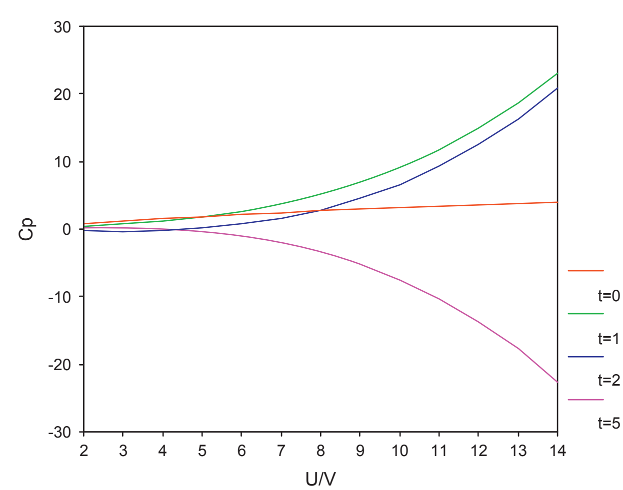

Fig. 18. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.023 and rotor solidity (σ) as 0.25.

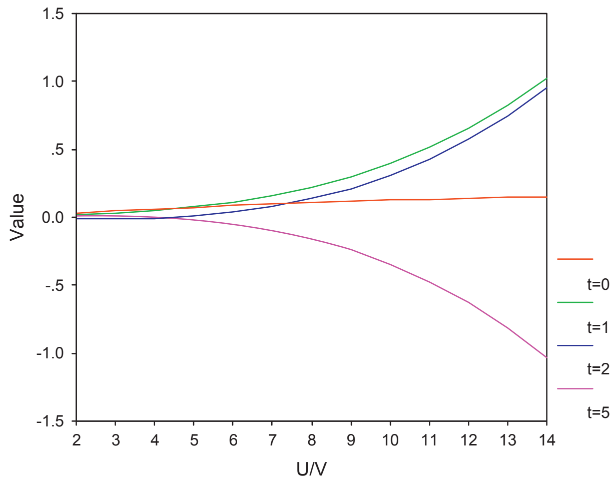

Fig. 19. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.025 and rotor solidity (σ) as 0.01.

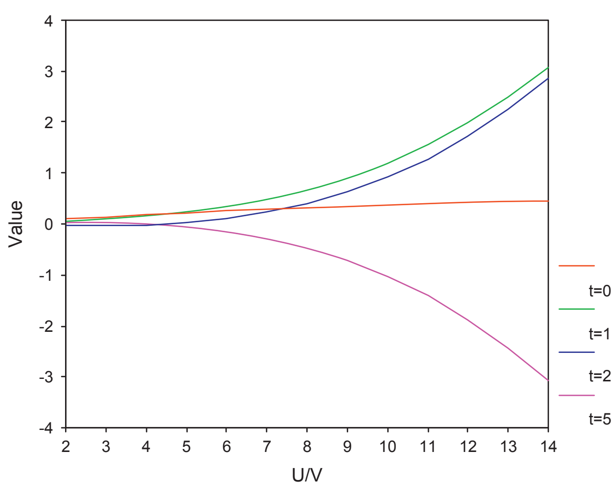

Fig. 20. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.025 and rotor solidity (σ) as 0.03.

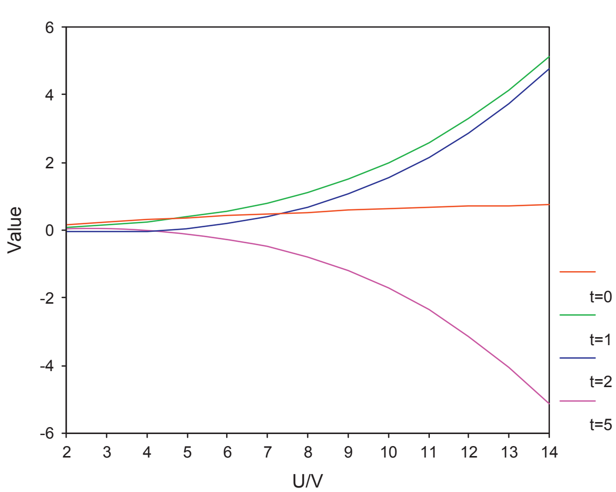

Fig. 21. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.025 and rotor solidity (σ) as 0.05.

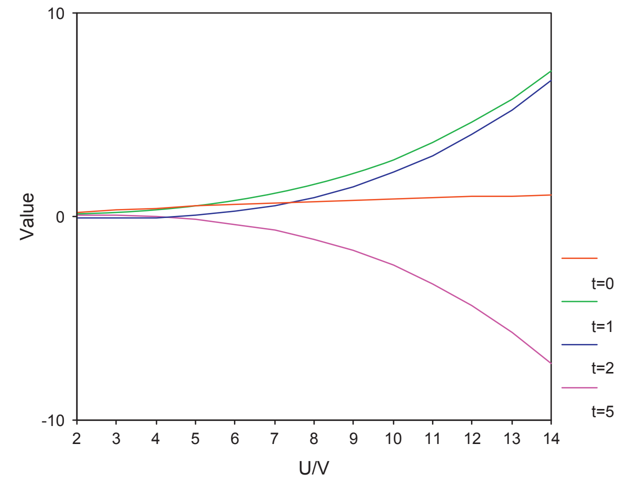

Fig. 22. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.025 and rotor solidity (σ) as 0.07.

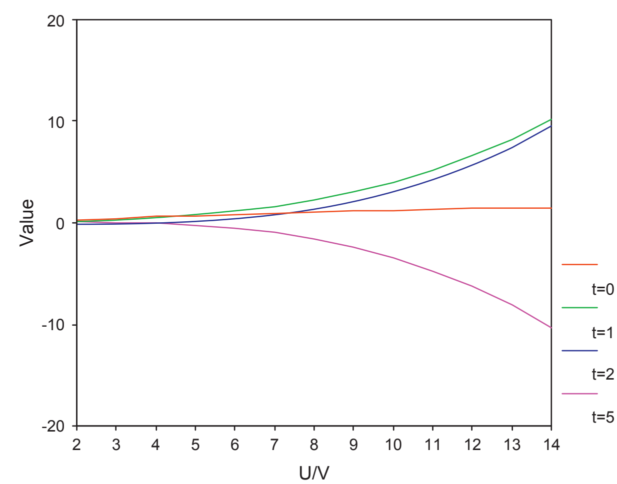

Fig. 23. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.025 and rotor solidity (σ) as 0.10.

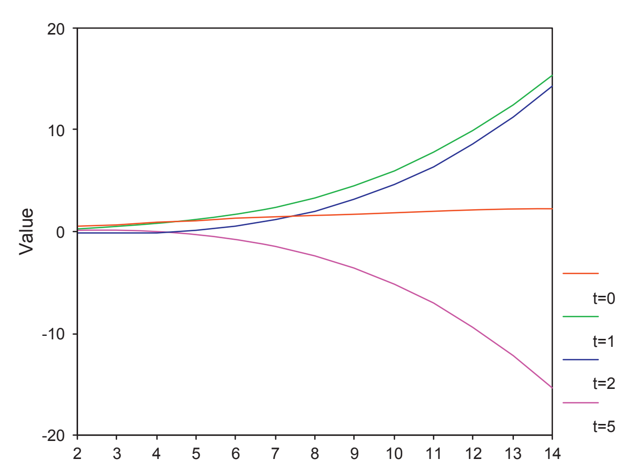

Fig. 24. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.025 and rotor solidity (σ) as 0.15.

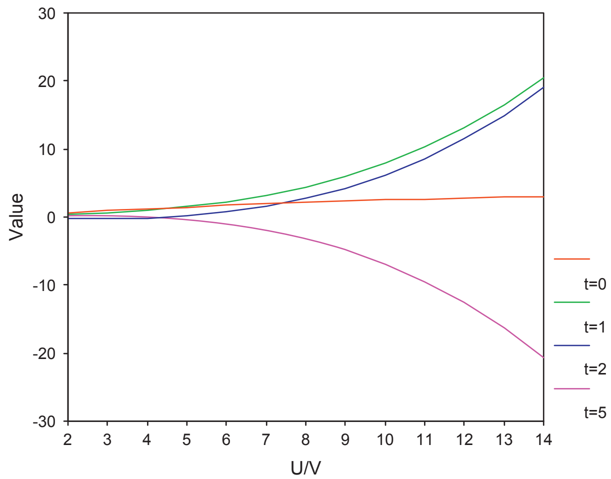

Fig. 25. Effect of power coefficient C_P with tip speed ratio (U/V)for different blade angle θ with (C_D/C_L) ratio of 0.025 and rotor solidity (σ) as 0.20.

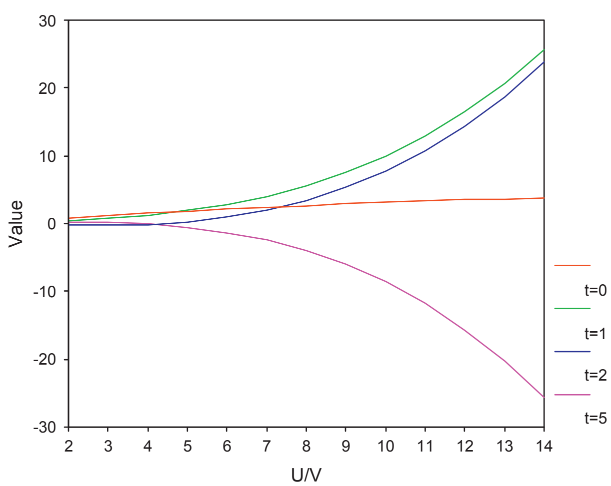

Fig. 26. Effect of power coefficient C_P with tip speed ratio (U/V)for different blade angle θ with (C_D/C_L) ratio of 0.025 and rotor solidity (σ) as 0.25

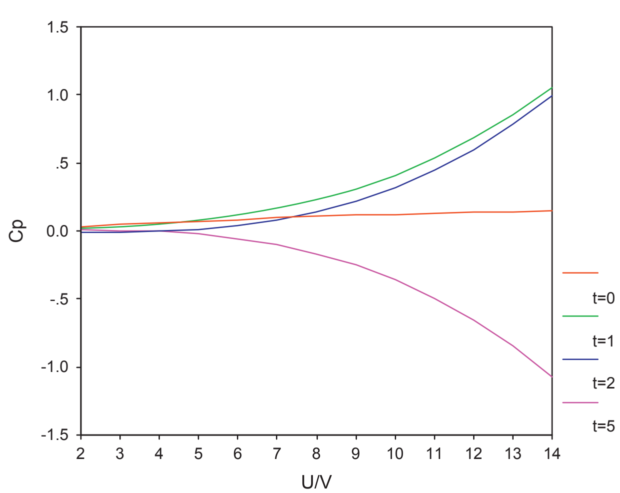

Fig. 27. Effect of power coefficient C_P with tip speed ratio (U/V)for different blade angle θ with (C_D/C_L) ratio of 0.026 and rotor solidity (σ) as 0.01.

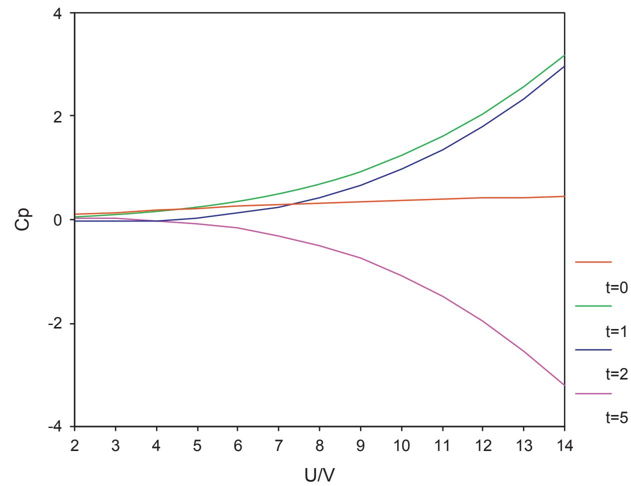

Fig. 28. Effect of power coefficient C_P with tip speed ratio (U/V)for different blade angle θ with (C_D/C_L) ratio of 0.026 and rotor solidity (σ) as 0.03.

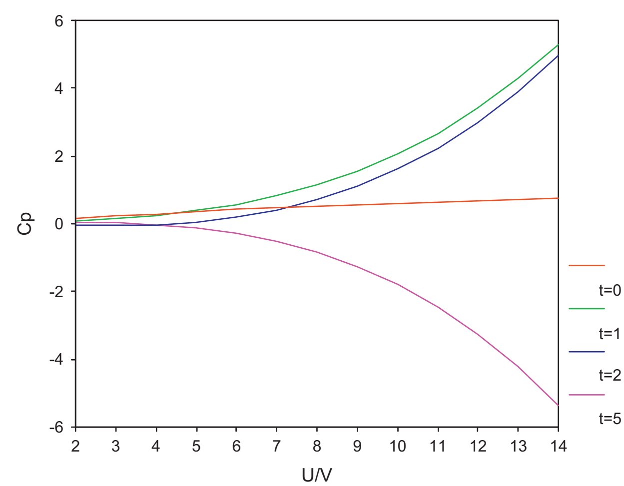

Fig. 29. Effect of power coefficient C_P with tip speed ratio (U/V)for different blade angle θ with (C_D/C_L) ratio of 0.026 and rotor solidity (σ) as 0.05

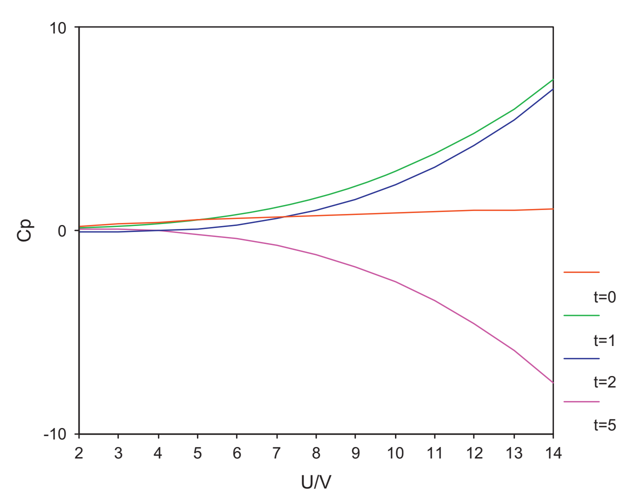

Fig. 30. Effect of power coefficient C_P with tip speed ratio (U/V)for different blade angle θ with (C_D/C_L) ratio of 0.026 and rotor solidity (σ) as 0.07.

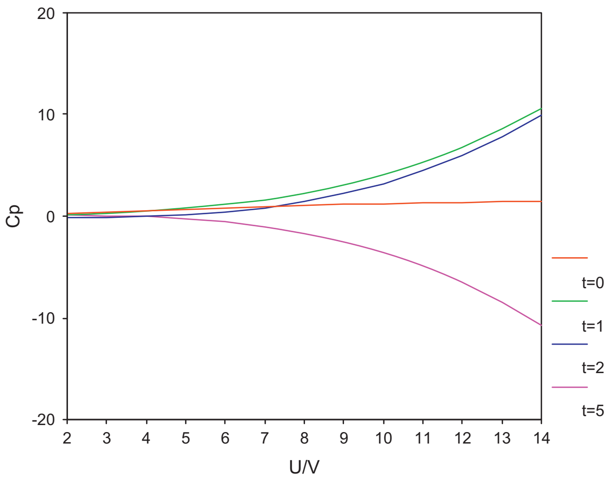

Fig. 31. Effect of power coefficient C_P with tip speed ratio (U/V)for different blade angle θ with (C_D/C_L) ratio of 0.026 and rotor solidity (σ) as 0.10.

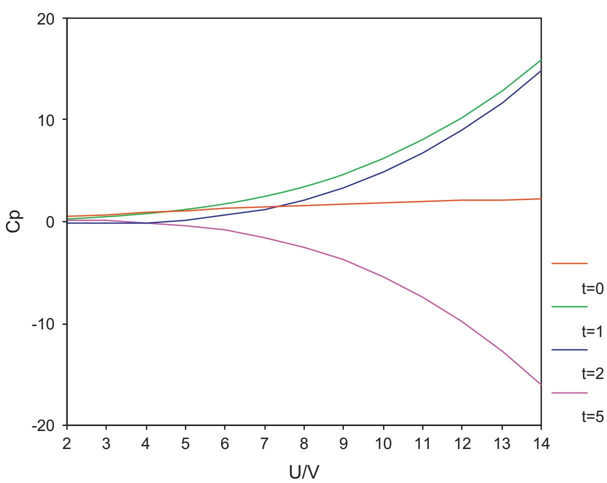

Fig. 32. Effect of power coefficient C_P with tip speed ratio (U/V)for different blade angle θ with (C_D/C_L) ratio of 0.026 and rotor solidity (σ) as 0.15.

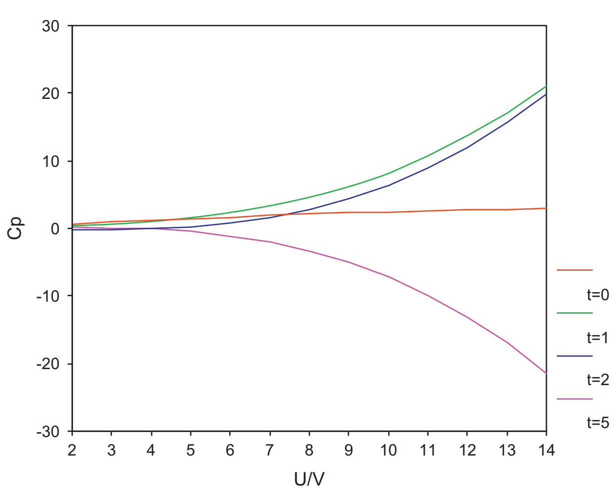

Fig. 33. Effect of power coefficient C_P with tip speed ratio (U/V)for different blade angle θ with (C_D/C_L) ratio of 0.026 and rotor solidity (σ) as 0.20.

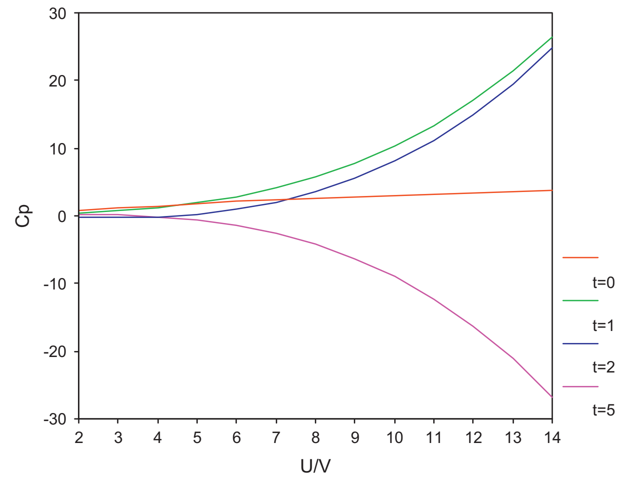

Fig. 34. Effect of power coefficient C_P with tip speed ratio (U/V)for different blade angle θ with (C_D/C_L) ratio of 0.026 and rotor solidity (σ) as 0.25.

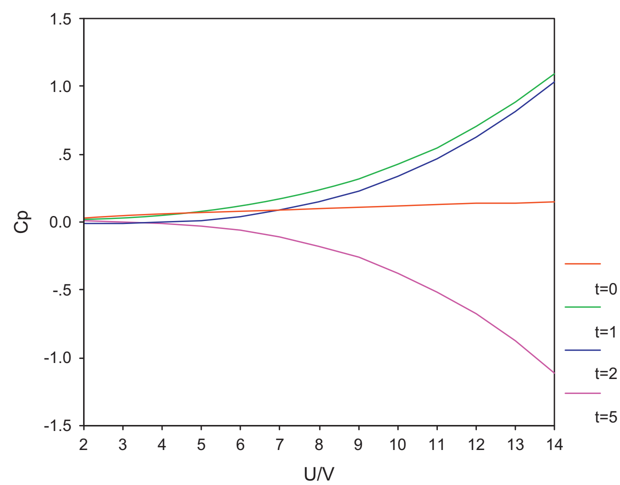

Fig. 35. Effect of power coefficient C_P with tip speed ratio (U/V)for different blade angle θ with (C_D/C_L) ratio of 0.028 and rotor solidity (σ) as 0.01

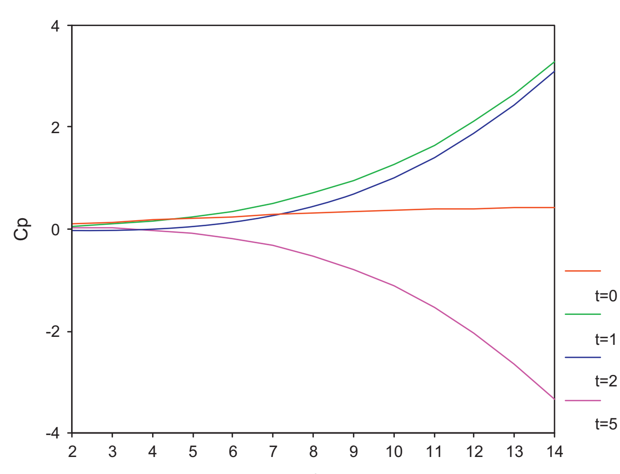

Fig. 36. Effect of power coefficient C_P with tip speed ratio (U/V)for different blade angle θ with (C_D/C_L) ratio of 0.028 and rotor solidity (σ) as 0.03.

equations, the solution of those equations was obtained with the help of Newton-Rapheson two variable method using MATLAB. The values of a, a′,(C_D/C_L), (U/V), σ and θ were substituted in the equations of C_P, C_T,C_Q using C – program. The optimized or efficient value were tabulated by finding coefficient of variation.

8. Results

Figs. 3–41 show the effect of power coefficient for different blade angle, tip speed ratio, ratio of coefficient of drag and coefficient of lift and blade solidity. The optimized set obtained is at blade solidity 0.15, angle of attack 5°, tip speed ratio 8 and ratio of drag coefficient to lift coefficient as 0.025.

9. Conclusion

The design of wind energy conversion systems is a very complex task and requires interdisciplinary skills, like civil, mechanical, electrical and electronics, geography, aerospace, environmental etc. An attempt has been made to discuss the important design aspects of WECs. The prospering future in wind turbine technology is a challenge for rotor blade design in order to enable an economic, reliable, safe and maintenance less production of wind energy. This paper also addresses the first significant step by defining the design parameters of developing an integrated dynamic, aerodynamic, and structural of wind turbine blades made of composite materials.

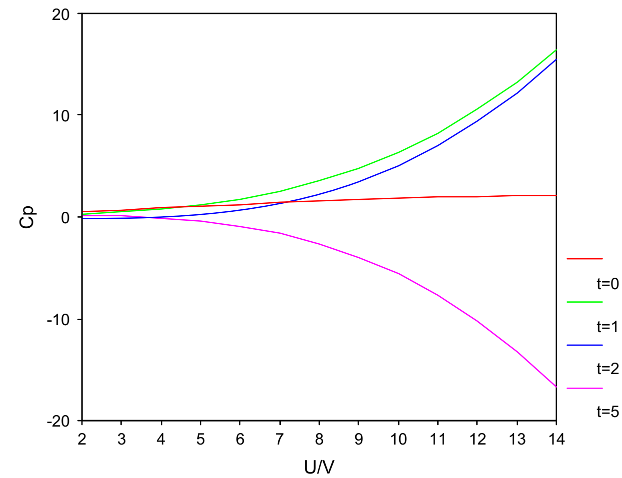

Fig. 39. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.028 and rotor solidity (σ) as 0.15.

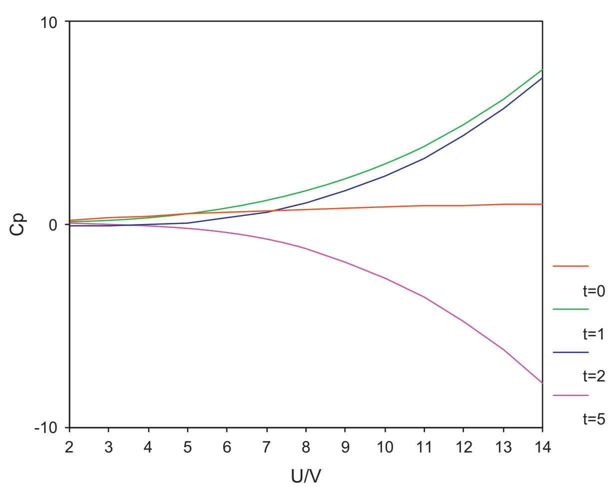

Fig. 37. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.028 and rotor solidity (σ) as 0.07.

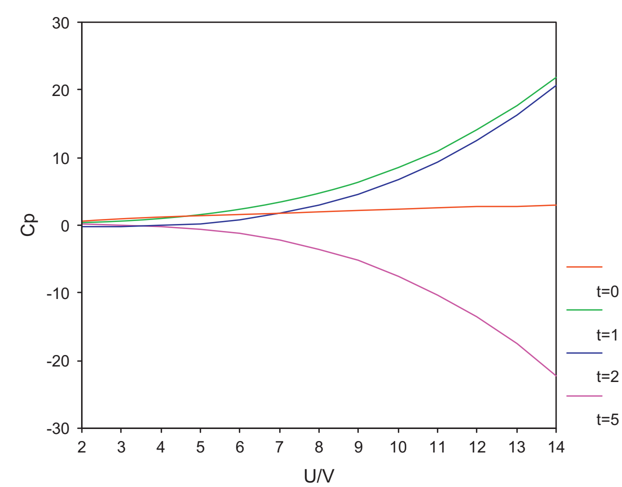

Fig. 40. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.028 and rotor solidity (σ) as 0.20.

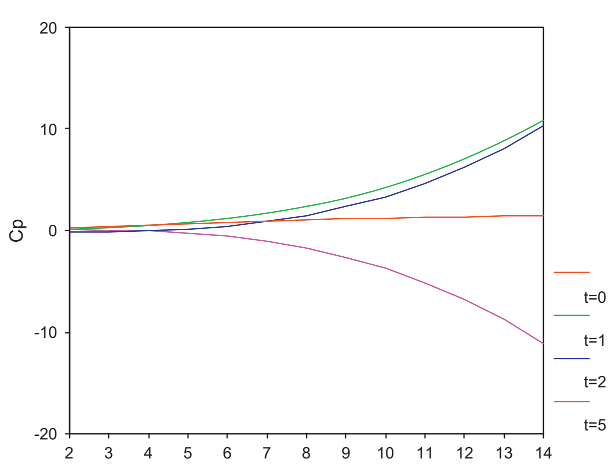

Fig. 38. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.028 and rotor solidity (σ) as 0.10.

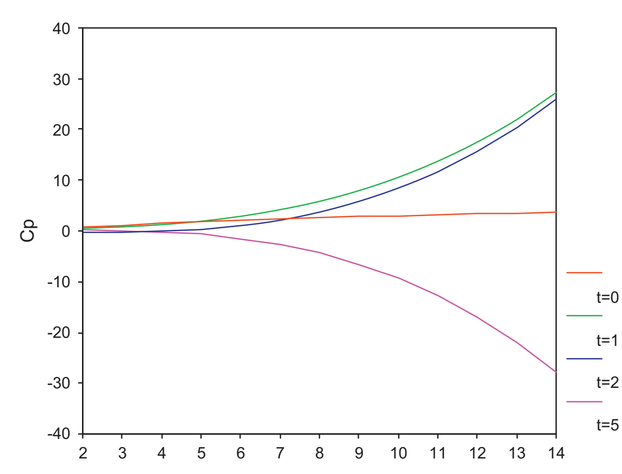

Fig. 41. Effect of power coefficient C_P with tip speed ratio $(U/V)$for different blade angle θ with (C_D/C_L) ratio of 0.028 and rotor solidity (σ) as 0.25.

Nomenclature

Symbols

A	swept area in m^2
P_a	total wind power available in Watts
P_e	electrical power output in Watts
m_w	mass flow rate of the wind in kg/s
V_m	mean velocity of wind in m/s
V_i	inlet wind velocity in m/s
V_o	outlet wind velocity in m/s
V_{ave}	average velocity in m/s
V_C	cut-in-speed in m/s
V_R	rated wind speed in m/s
V_F	furling speed in m/s
P_{out}	power recovered from the wind in Watts
$P_{out.max}$	maximum power that can be drawn from the wind in Watts
E_a	available wind energy
V_m	mean wind speed
E_{as}	available energy flux
T	time period
P_e	electrical power output in Watts
C_p	power coefficient
I	current in amps
V	voltage in volts
R	maximum rotor radius in m
P_s	Shaft power output in Watts
U_o	upstream undisturbed wind speed in m/s
C_L	aerodynamic lift coefficient
C_D	aerodynamic drag coefficient
C_M	power performance of a wind turbine
c	chord of aerodynamic profile
L	lift force
D	drag force
F_M	moment force
I	inclination angle
i	incidence angle

Greek Symbols

α	pitch angle
θ	angle of attack
λ	tip speed ratio
ω	angular rotor speed in rad/s
μ	Hellmann coefficient
η_g	generator efficiency
η_m	mechanical efficiency
η_a	alternator efficiency
ρ	density of air in kg/m^3

Abbreviations

GRP	Glass fiber Reinforced Plastics
NACA	National Advisory Committee of Aeronautics
HAWT	Horizontal Axis Wind Turbine
VAWT	Vertical Axis Wind Turbine
CSCF	Constant Speed Constant Frequency
VSCF	Variable Speed Constant Frequency
VSVF	Variable Speed Variable Frequency
TSR	Tip Speed Ratio
O & M	Operation & Maintenance cost
WECS	Wind Electric Conversion System
IEC	International Electrotechnical Commission
SEIG	Self- Excited Induction Generators

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